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Applied Research in Statistics - Mathematics - Operations Research

FACTOR SCREENING IN SIMULATION: PERFORMANCE OF A TWO-STAGE RANDOM BALANCE/PLACKETT-BURMAN PROCEDURE

by

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I. INTRODUCTION

Simulation models that involve many factors (i.e., input variables) necessitate large experimental programs. Often, though, it is expected that only a relatively small number of the factors have an appreciable effect on the response (i.e., output variable). In order to reduce costs by concentrating the major experimental effort on the most important factors, a frequent desire is to screen the factors in the hope of selecting the important ones for further, more detailed study in subsequent experimentation. In such a situation, the simulation user would like to conduct a preliminary experiment aimed at determining the size and composition of the subset of important factors.

Although this report places factor screening in a simulation context, screening experiments can arise in virtually any field of scientific research. In fact, much of the existing screening methodology is a result of research in the arena of industrial experimentation. Overall, however, the problem has yet to be resolved satisfactorily or studied systematically. Thus, no definitive guidelines for selecting a screening strategy have been developed, although a number of general suggestions have been made. [See Kleijnen (1975) for a review.]

To a large extent, choosing a screening procedure is governed by the number of simulation runs available for screening. When affordable, screening strategies based on standard orthogonal experimental designs are recommended. However, these designs require more runs than factors to be screened, a luxury seldom provided in the simulation environment because of budget and running time limitations. Thus, screening methods based on nonstandard designs which allow for fewer runs than factors to be screened are usually of greater practical interest.

In a previous Desmatics technical report, Smith and Mauro (1980) reviewed the major classes of screening designs, discussed possible performance criteria for systematically evaluating screening methods, and proposed two screening strategies for further research, namely, two-stage group screening and a random balance/Plackett-Burman (RB/PB) two-stage combination strategy. Both proposed strategies are intended for situations in which the available number of computer runs is less than the number of factors to be screened. More recently, Mauro and Smith (1980) formally examined and reported the performance of the two-stage group screening method. This report presents a parallel analysis for the *RB/PB screening method*.

II. PRELIMINARY DISCUSSION

In screening experiments a relatively small number of factor levels is generally employed. Usually two levels, designated high (+1) and low (-1), of each factor are sufficient. Consequently, most screening designs are two-level designs. However, in a two-level experiment it is appealing to consider only designs which have an equal number of runs at the high and at the low levels of a factor. That is, for a design employing N (even) runs, each column of the design matrix would consist of N/2 +1's and N/2 -1's. Estimates of factor effects are then unconfounded with the overall mean effect, which can be represented by a column of N +1's.

In a random design, a random sampling process is used to choose all or some of the elements of the design matrix. In generating the design matrix, various sampling schemes can be employed. However, as indicated, for a two-level situation it is desirable to balance the design. Consider, therefore, a design matrix in which the assignment of +1's and -1's in each column is made randomly so that all possible configurations of N/2 +1's and N/2 -1's (there are $C^{N}_{N/2}$ in all) are equally likely, with each column receiving an independent randomization. An experimental design that is a realization of such a sampling plan is called a random balance design. Note that N can be determined independently of the number of factors to be screened. This is not true of more orthodox factorial designs.

Random balance designs have generated much heated discussion as to their merits and demerits [e.g., Bunde (1959), Satterthwaite (1959), and Youden, Kempthorne, Tukey, Box, and Hunter (1959)]. The main objection is that they confound factors to a random degree. Furthermore, there are no specific techniques for analyzing random balance data. Typically, each factor is considered separately, and some standard statistical analysis is applied to test for the presence of factor effects. Anscombe (1959) suggested using the ANOVA F-test, Welch's Randomization Test, or Tukey's Randomization Test. These procedures all admit to a relatively quick and simple statistical analysis. Conceivably, more specialized techniques could be devised and employed, but at the expense of analysis simplicity. In this report, a standard F-test applied separately to each factor will be adopted as the method of analysis for random balance data.

To provide the same statistical basis to compare and assess screening strategies, it is necessary to assume a common model to underlie the simulation responses. In screening, it generally suffices to assume the first-order model given by

$$y_i = \beta_0 + \sum_{j=1}^{K} \beta_j x_{ij} + \varepsilon_i \tag{2.1}$$

where

(1) y_i is the value of the response in the $i^{\underline{th}}$ simulation run,

(2) K is the total number of factors to be screened, each of which is at two levels,

(3) x_{ij} is +1 or -1 provided the $j^{\underline{th}}$ factor is set at its high or low level, respectively, for the $i^{\underline{th}}$ simulation run,

(4) β_j is the (linear) effect of the $j^{\underline{th}}$ factor,

and (5) the error terms, ε_i, are independent with common distribution $N(0,\sigma^2)$.

For a given set of values for β_1, $\beta_2, \ldots, \beta_K$, and σ in model (2.1), the performance of a screening method can then be evaluated. However, to keep this study manageable the following additional conditions[1] will be imposed:

(2') there are a total of K factors to be screened, $k \geq 1$ (unknown) of which are active (i.e., have a nonzero effect) and $(K - k)$ of which are inactive (i.e., have a zero effect),

(4') all active factors have the same absolute linear effect, $\Delta > 0$, i.e.,

$$|\beta_j| = \begin{cases} \Delta, & \text{if the } j^{\underline{th}} \text{ factor is active} \\ 0, & \text{if the } j^{\underline{th}} \text{ factor is inactive,} \end{cases}$$

and (5') the simulation response is observed without random error (i.e., $\sigma=0$).

By (4') all active factors are regarded symmetrically. Furthermore, (5') is equivalent to assuming that Δ is so large compared with σ that σ can be effectively ignored. A forthcoming report will examine the performance of the RB/PB strategy when this assumption is relaxed.

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[1]These are the same assumptions under which Mauro and Smith (1980) examined the two-stage group screening method.

III. THE RB/PB STRATEGY

The first stage of the RB/PB strategy is a K factor random balance (RB) design employing N (even) runs, while the second is a Plackett-Burman[1] (PB) design follow-up experiment. A factor reaches stage two only if it has a significant effect in stage one. Because PB designs are orthogonal, the second stage will filter out any confounding between the factors carried over from the RB design. To be classified as active, a factor must test significant in the PB design, which can be analyzed by the usual analysis of variance procedures for factorial experiments. Factors not reaching stage two, however, should be held at a constant level so that the second-stage estimate of a factor effect is not biased by any effects of active factors not reaching this stage. [cf. Montgomery (1979).] The number of second stage runs, M, will depend on the number of factors, S, carried over from stage one. In general, the smallest PB design having at least S+1 runs is chosen. Thus, if B(x) denotes the smallest integer larger than x that is a multiple of four, then M = B(S). Accordingly, the total number of runs, R, that the two stages require will be N + M = N + B(S). Note that although N must be specified prior to experimentation, M, and consequently R, is random.

As Watson (1961) pointed out, the objectives of a factor screening strategy are:

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[1]Plackett-Burman designs are two-level, orthogonal, fractional factorial designs for studying up to (4m-1) factors in 4m runs. See Plackett and Burman (1946).

(a) to detect as many of the active factors as possible,

(b) to declare active as few inactive factors as possible,

and (c) to achieve these aims with as few runs as possible.

Therefore, the extent to which the RB/PB strategy, or any other screening strategy, satisfies the above objectives is a measure of its performance as a screening strategy. However, since objectives (a) and (b), which deal with factor classification, and objective (c), which deals with testing cost, conflict, some balancing of objectives will be required to compare competing strategies.

Because of the additive structure of model (2.1) and the resolution of PB designs, a factor that reaches stage two will, under the assumption of no random error ($\sigma=0$), be classified correctly. Thus, under the assumptions made here, objective (b) is trivially satisfied in the RB/PB strategy since no inactive factor will be misclassified. Performance evaluation can focus, therefore, on objectives (a) and (c). Moreover, the number of active factors classified correctly, A, will correspond to the number of active factors that test significant in stage one.

As a measure of efficiency for detecting the active factors, define

$$E(F) = E(A)/k \quad . \tag{3.1}$$

E(F) is normalized so that $0 \leq E(F) \leq 1$; efficiency closer to one indicates better performance on the average. Regarding the total runs that a strategy requires,

$$E(Q) = E(R)/B(K) \tag{3.2}$$

can be used as a measure of the expected relative testing cost of a screening strategy (relative to the runs required by a PB design for K factors.) A smaller value of E(Q) indicates better performance on the average. It is imperative, however, that both (3.1) and (3.2) be considered jointly in assessing the overall performance of a RB/PB strategy. In many ways the problem is akin to the testing of a statistical hypothesis in which the sample size is desired small, but the corresponding power is desired large.

Only if one RB/PB strategy has both a smaller E(Q) and a larger E(F) than another can the first be said to be definitely better than the second. For example, in Figure 1 it is clear that strategy A is superior to B and C. However, in comparing A with D, superiority depends on possible trade-offs in E(F) and E(Q) the analyst is willing to make in a given screening application. A cost function could, of course, be defined in terms of E(F) and E(Q) to compare these two strategies.

For simplicity, suppose that the K separate F-tests performed in stage one are each conducted at the same level of significance, α. Of interest, therefore, is how α and N affect performance, as defined by (3.1) and (3.2). To establish a notation, let $RP(c,\alpha)$ denote the RB/PB strategy in which $N = cK$, $0 < c < 1$, and let I denote the number of inactive factors that test significant in stage one. Since $S = A + I$ and $B(x) \approx x + 2.5$, it follows that[1],

$$E(R) \approx cK + E(A) + E(I) + 2.5 \quad . \qquad (3.3)$$

Thus, to evaluate (3.1) and (3.2) it suffices to evaluate E(A) and E(I), both of which depend on the testing characteristics of the K separate

[1] Note that $|B(x) - (x + 2.5)| \leq 1.5$; hence, the approximation in (3.3) can differ from E(R) by at most 1.5 .

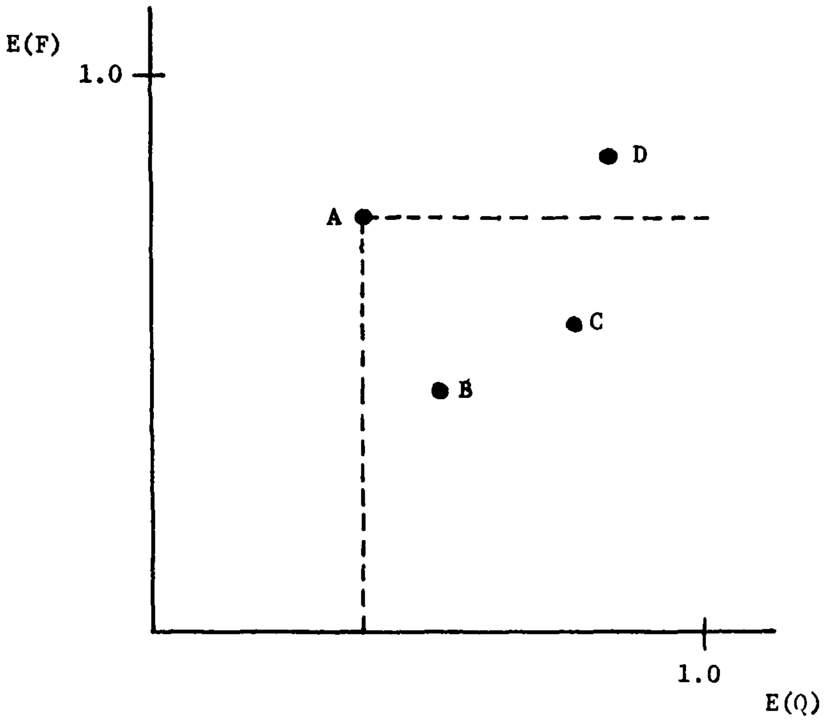

Figure 1: Plot of **E(F)** versus E(Q) for four hypothetical RB/PB strategies.

F-tests in the RB design.

In a random balance design employing N (even) runs, let $\underline{X} = [\underline{x}_0, \underline{x}_1, \underline{x}_2, \ldots, \underline{x}_K]$ denote the N x (K+1) design matrix where $\underline{x}_0$ is a (Nx1) vector consisting entirely of +1's and $\underline{x}_j$ ($j \geq 1$) is a (Nx1) vector consisting of a random arrangement of N/2 +1's and N/2 -1's. Assuming that $\sigma=0$ in model (2.1), $\underline{y} = \underline{X}\underline{\beta}$ where $\underline{\beta}$ is a (K+1) x 1 vector of unknown parameters and $\underline{y}$ is a (Nx1) vector of responses.

The simple least squares estimator of β_j ($j \geq 1$) is given by

$$\hat{\beta}_j = (\bar{y}_{+j} - \bar{y}_{-j})/2 \tag{3.4}$$

where $\bar{y}_{+j}$ ($\bar{y}_{-j}$) is the average value of the response over the N/2 runs at the high (low) level of the $j^{\underline{th}}$ factor. In matrix terms

$$\hat{\beta}_j = (\underline{x}_j'\underline{y})/N = \underline{x}_j'(\sum_{i=1}^{K} \beta_i \underline{x}_i)/N \quad . \tag{3.5}$$

Thus,

$$E(\hat{\beta}_j) = (1/N) \sum_{i=1}^{K} \beta_i E(\underline{x}_j'\underline{x}_i) \quad , \tag{3.6}$$

and $$V(\hat{\beta}_j) = (1/N^2) \sum_{i=1}^{K} \beta_i^2 V(\underline{x}_j'\underline{x}_i) \quad , \tag{3.7}$$

where equation (3.7) makes use of the fact that $Cov(\underline{x}_j'\underline{x}_i, \underline{x}_j'\underline{x}_m) = 0$ when $i \neq m$. To evaluate these moments a brief digression is taken.

If Z is a discrete random variable such that

$$P(Z=z) = \begin{cases} \binom{r}{z}^2 \Big/ \binom{2r}{r} & z = 0, 1, 2, \ldots, r \\ 0 & \text{otherwise,} \end{cases}$$

write $Z \sim H(r)$. The H(r) family is a unimodal, symmetric subfamily of the hypergeometric family of distributions. Furthermore, if $Z \sim H(r)$, then $E(Z) = r/2$ and $V(Z) = r^2/4(2r - 1)$. Now a key result is that, for $i \neq j$, $(\underline{x}_i{}'\underline{x}_j + N)/4 \sim H(N/2)$. Hence,

$$E(\hat{\beta}_j) = \beta_j \ , \tag{3.8}$$

$$V(\hat{\beta}_j) = (W - \beta_j^2)/(N - 1) \ , \tag{3.9}$$

$$\text{and} \quad Cov(\hat{\beta}_i, \hat{\beta}_j) = \beta_i\beta_j/(N - 1) \ , \ i \neq j, \tag{3.10}$$

where $W = \sum_{j=1}^{K} \beta_j^2$. Of course, only active factors make a nonzero contribution to W. When $i = j$, $\underline{x}_i{}'\underline{x}_j = \underline{x}_i{}'\underline{x}_i = N$.

The simple least squares estimator of β_j, $\hat{\beta}_j$, is therefore an unbiased estimator. However, an obvious drawback of analyzing factors separately is that the squared effects of the ignored factors are absorbed into the variance of $\hat{\beta}_j$. Hence, even when the responses are observed without experimental error, $V(\hat{\beta}_j)$ may be unduly large.

Because each factor is at two levels, the standard F-test to test H_0: $\beta_j = 0$ vs H_1: $\beta_j \neq 0$ is equivalent to a simple two-sample t-test between the high (+1) and the low (-1) levels of the $j^{\underline{th}}$ factor. The corresponding test statistic, t_j, is given by

$$t_j = \hat{\beta}_j / \ [SSE_j/N(N - 2)]^{\frac{1}{2}} \tag{3.11}$$

where SSE_j is the familiar analysis of variance notation for the error sum of squares of factor j. Computationally,

$$SSE_j = \sum_H (y_i - \bar{y}_{+j})^2 + \sum_L (y_i - \bar{y}_{-j})^2 \tag{3.12}$$

where the first (second) summation is taken over the N/2 observations at the high (low) level of the $j^{\underline{th}}$ factor. Other useful identities include

$$SSE_j = \sum_{i=1}^{N} y_i^2 - N\hat{\beta}_j^2 - N\bar{y}^2 , \qquad (3.13)$$

$$SSE_j = N \left[\sum_{i=1}^{K} \beta_i \hat{\beta}_i - \hat{\beta}_j^2 \right] , \qquad (3.14)$$

and

$$SSE_j = \underline{v}'\underline{v} - N\hat{\beta}_j^2 \qquad (3.15)$$

where $\bar{y}$ is the overall mean of the responses and $\underline{v} = \sum_{i=1}^{K} \beta_i \underline{x}_i$.

Since H_1 is a two-sided alternative, it is reasonable to reject H_0 if the observed value of $|t_j|$ equals or exceeds the upper $100(1 - \alpha/2)$ percentage point of a t distribution having (N-2) degrees of freedom. Denote this critical value by $t(N-2; \alpha/2)$. However, it should be emphasized that although t_j has the general form of a t statistic, the distribution of t_j under H_0 (i.e., when factor j is inactive) is not a t distribution. Consequently, the test of H_0: $\beta_j = 0$ vs H_1: $\beta_j \neq 0$ outlined here is not an exact testing procedure in that the true significance level of the test may differ from α.

In the following section, results based on the empirical distribution of t_j, obtained by Monte Carlo, are compared with those based on an approximating t distribution. As will be seen, t_j is, in general, fairly well-approximated by a central t variable under H_0 and by a noncentral t variable under H_1.

IV. PERFORMANCE EVALUATION OF THE RB/PB STRATEGY

Define

$$V_j = \begin{cases} 1, \text{ if } H_0\text{: } \beta_j = 0 \text{ is rejected} \\ 0, \text{ if } H_0\text{: } \beta_j = 0 \text{ is not rejected,} \end{cases}$$

and

$$d_j = \begin{cases} 1, \text{ if factor } j \text{ is active} \\ 0, \text{ if factor } j \text{ is inactive.} \end{cases}$$

Observe that

$$E(A) = \sum_{j=1}^{K} d_j \, E(V_j) \, , \tag{4.1}$$

and

$$E(I) = \sum_{j=1}^{K} (1 - d_j) \, E(V_j) \, . \tag{4.2}$$

Accordingly, to evaluate E(A) and E(I), and subsequently E(F) and E(Q), it suffices to calculate $E(V_j)$ for all j. Note, however, that

$$E(V_j) = P(V_j = 1) = P \, \{|t_j| \geq t(N-2;\, \alpha/2)\}.$$

If the exact sampling distribution of t_j was known, then this probability could be evaluated directly and precisely. Unfortunately, the distribution of t_j is intractable. Performance evaluation must rely, therefore, on an approximating distribution or on Monte Carlo estimation.

Motivated by large-sample properties of t_j, it is possible to obtain a fixed sample size approximation to the distribution of t_j. This approximation states that for fixed N, t_j has an approximate noncentral t distribution with N - 2 degrees of freedom and noncentrality parameter

$$\delta_j = N^{\frac{1}{2}}\beta_j / [W - \beta_j^2]^{\frac{1}{2}}. \qquad (4.3)$$

Hence, if $T_{N-2}(\delta_j)$ denotes a random variable having such a distribution, then

$$E(V_j) \approx P\{|T_{N-2}(\delta_j)| \geq t(N-2;\ \alpha/2)\}. \qquad (4.4)$$

Denote the probability in (4.4) by $\psi(\delta_j)$. Of course, when factor j is inactive, the corresponding probability equals α, since $T_{N-2}(0)$ has a central t distribution. Furthermore, when all active factors have the same absolute linear effect, $\psi(\delta_j)$ is the same for each active factor because of symmetry. Thus

$$E(A) \approx k\psi(\delta) \qquad (4.5)$$

and $$E(I) \approx (K - k)\alpha \qquad (4.6)$$

where $\delta = [N/(k - 1)]^{\frac{1}{2}}$.

The quantity $\psi(\delta)$ is simply the power of the test for detecting an active factor. Substituting (4.5) and (4.6) into (3.1), (3.2) and (3.3) yields

$$E(F) \approx \psi(\delta) \qquad (4.7)$$

and $$E(Q) \approx [cK + k\psi(\delta) + (K - k)\alpha + 2.5] / B(K). \qquad (4.8)$$

In order to examine the quality of the approximations provided by the noncentral t distribution, a Monte Carlo computer program was used to obtain empirical estimates for comparison in a number of cases.

On the whole, the approximations performed quite well, even for relatively small values of N. As an illustration, Tables 7 and 13 in the Appendix contain the Monte Carlo estimates and the approximations provided by the noncentral t distribution, respectively, when K = 120 and k = 10. The entries in these tables correspond to the 126 RP(c,α) strategies defined by c = .1, .2, ..., .8, .9 and α = .05, .10, ..., .65, .70 . As can be seen, the results agree fairly closely even when c = .1 .

In all, performance data for twelve specific cases of (K,k) is tabled in the Appendix. For each of three K values (60, 120, and 240) examined, four values of k (k = p*K, where p* = 2/60, 3/60, 5/60, and 8/60) were studied. However, because the Monte Carlo procedures became prohibitively expensive for large N and K, it was not feasible to simulate over the full range of c for all cases. Thus, the performance tables in the Appendix (except for K = 120 and k = 10) consist of both Monte Carlo estimates (for smaller c) and approximated values (for larger c) of E(A), E(R), E(F), and E(Q). Each table caption indicates the source of the entries.

In general, E(F) is larger and E(Q) is smaller for a smaller p*. Thus, as intuition would suggest, there is more to be gained from the RB/PB screening strategy when the proportion of active factors is smaller. Furthermore, for fixed p*, both E(F) and E(Q) increase as either α or c increases. An increase in the significance level, α, results in a less stringent rejection rule so that on the average more factors, both inactive and active, reach stage two, hence increasing E(F) and E(Q). An increase in the number of first stage runs, c, increases directly both E(Q) and the power of the test, hence E(F).

For fixed c, E(F) increases from 0 to 1 and E(Q) increases from cK/B(K) to 1 + (cK)/B(K) as α increases over the interval [0,1]. Therefore, no matter what value of c is decided upon, by adjusting α, E(F) can be increased or E(Q) can be decreased at the expense, of course, of increasing E(Q) or decreasing E(F), respectively. Consequently, in practice, selecting the c and α to actually employ will depend on performance trade-offs the analyst is willing to make.

To illustrate how the selection of a RP(c,α) strategy might occur, suppose K = 240 factors have been suggested for analysis, but only about 12 are expected to have an appreciable effect on the response. Using the data of Table 10 (K = 240 and k = 12), contour plots of E(F) and E(Q) as functions of c and α were constructed to graphically illustrate the available trade-offs. These plots were overlaid and are exhibited in Figure 2. Because of budget considerations, suppose further that it is desired to employ the RP(c,α) strategy that maximizes E(F) subject to $E(Q) \leq .5$. For all practical purposes, as is apparent from Figure 2, any RP(c,α) strategy corresponding to the segment AB should suffice. For these strategies E(F) is roughly .85.

As a final note, it is instructive to rewrite δ and (4.8) as

$$\delta \approx (c/p^*)^{\frac{1}{2}} \tag{4.9}$$

and

$$E(Q) \approx c + p^*\psi(\delta) + \alpha(1 - p^*) + (2.5/K) \tag{4.10}$$

where (4.10) follows from (4.8) by dividing by K instead of B(K). Recall that B(K) and K can differ by at most four.

Equations (4.9) and (4.10) together with (4.6) show basically how c, α, and p^* will affect performance. For instance, increasing δ, which is equivalent to increasing c or decreasing p^*, increases $\psi(\delta)$, i.e., E(F).

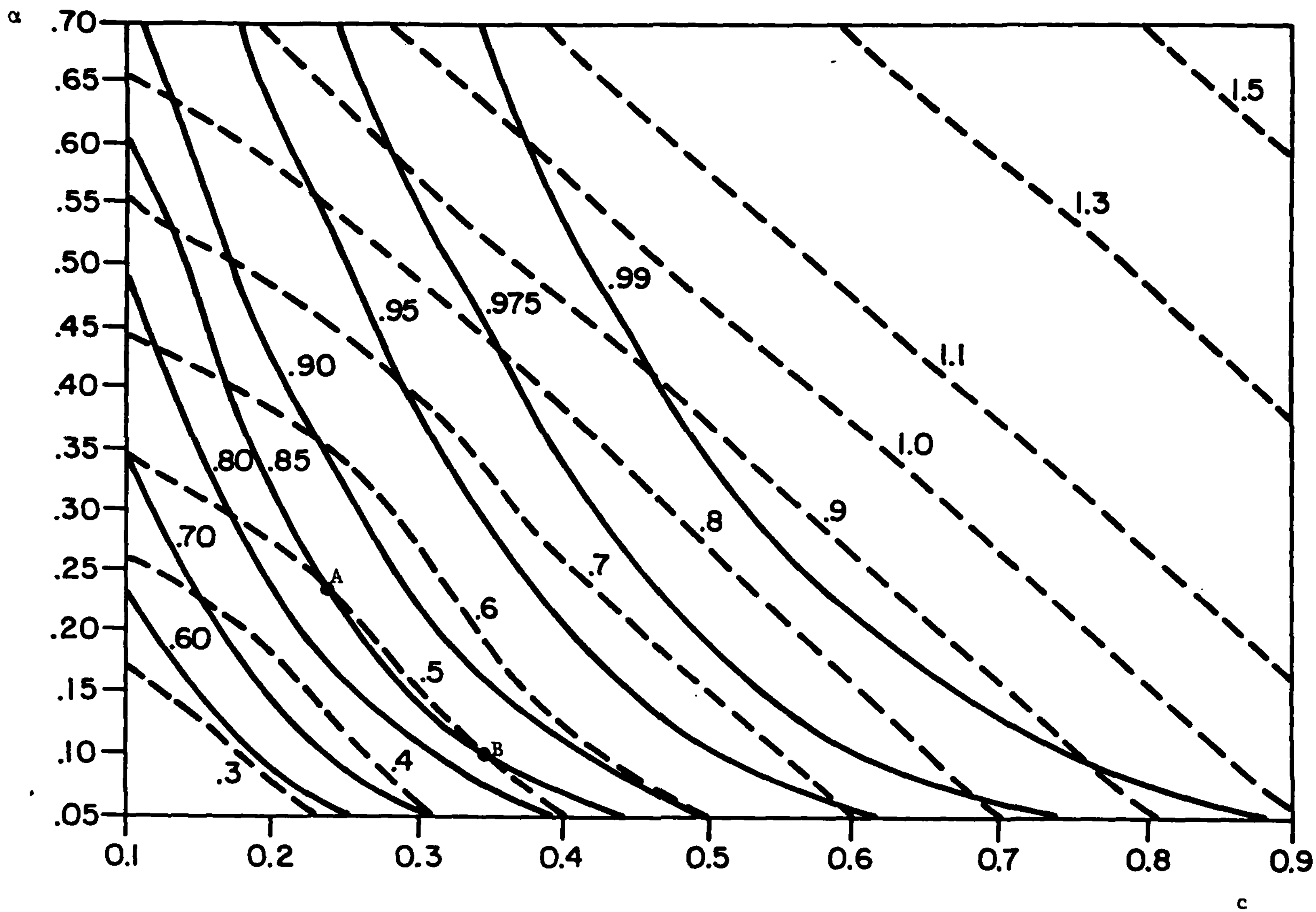

Figure 2: Overlay of Contour Plots of E(F), Denoted by Solid Line, and E(Q), Denoted by Dashed Line, as Functions of c and α When K = 240 and k = 12.

Equation (4.8) also illustrates that E(F) depends on K and k essentially only through p*. This is further evidenced by the data in the Appendix. Thus, the contour plot of E(F) in Figure 2 could be used as a reasonable approximation for other K and k cases in which $k/K = .05$. Finally, as already noted, increasing c or α will increase E(Q), although the effect of increasing p* is not directly discernible. Data in the Appendix indicates, however, that E(Q) increases with p*.

V. CONCLUDING DISCUSSION

This report evaluates, in cases of no random error (or equivalently, where the effects of active factors are known to be large enough to ignore random error), the factor classification and testing cost characteristics of the RB/PB screening strategy. Because these are conflicting objectives, an overall performance criterion for comparing different RB/PB strategies must consider these characteristics jointly. Ultimately, in a given situation it is the simulation user who must decide on the trade-offs to be made. Accordingly, the results of this report should serve as a practical guide in decisions about the possible use and choice of an RB/PB strategy.

In section IV, comparison with Monte Carlo estimates showed that the noncentral t distribution provided an adequate approximation to $E(V_j)$ when all active factors have the same absolute effect, Δ. Although no empirical estimates were calculated in the case of an arbitrary $\underline{\beta}$, it seems reasonable that $\psi(\delta_j)$ will still provide an adequate approximation to $E(V_j)$. However, active factors should probably not be treated symmetrically since their magnitudes may differ. In such cases, $E(F)$ could be defined as

$$E(F) = \sum_{j=1}^{K} d_j \, |\beta_j| \, E(V_j) / \sum_{j=1}^{K} |\beta_j| \qquad (5.1)$$

where d_j is defined in section IV. Note that $0 \leq E(F) \leq 1$ and reduces to (3.1) when all active factors have the same absolute effect. Performance, $E(F)$ and $E(Q)$, of the $RP(c,\alpha)$ strategy, therefore, can be easily extended to an arbitrary $\underline{\beta}$ by letting $E(V_j) \approx \psi(\delta_j)$ and substituting this approximation into (4.1), (4.2), (3.2), (3.3), and (5.1).

As a by-product, the results of this report help to shed some light on the controversy surrounding random balance experimentation. For instance, let RB(c,α) denote the screening strategy based solely on a random balance design having $N = cK$ runs. Suppose again that each factor is tested for importance separately with a standard F-test at the α level of significance. In the case of no random error where all active factors have the same absolute effect,

$$E\{F|RB(c,\alpha)\} = E\{F|RP(c,\alpha)\} \approx \psi(\delta)$$

$$E\{Q|RB(c,\alpha)\} = cK/B(K),$$

$$\text{and} \qquad E\{J|RB(c,\alpha)\}/(K - k) \approx 1 - \alpha$$

where J denotes the number of inactive factors identified correctly.

Note that the classification efficiency of the RB(c,α) strategy for detecting the active factors is the same as that of the RP(c,α) strategy, but that the relative testing cost of the RB(c,α) strategy is smaller since there is no follow-up stage. Of course, deleting the PB second stage introduces the possibility of classifying as active an inactive factor. In this case, overall performance will require evaluation against all three of the screening objectives listed in section III.

VI. REFERENCES

Anscombe, F. J. (1959). "Quick Analysis Methods for Random Balance Screening Experiments," Techometrics, Vol. 1, p. 195.

Budne, T. A. (1959). "The Application of Random Balance Designs," Techometrics, Vol. 1, p. 139.

Kleijnen, J. P. C. (1975). "Screening Designs for Poly-Factor Experimentation," Techometrics, Vol. 17, p. 487.

Mauro, C. A., and Smith, D. E. (1980). "An Examination of the Performance of Two-Stage Group Screening for Use in Computer Simulation," Technical Report No. 113-3, Desmatics, Inc.

Montgomery, D. C. (1979). "Methods for Factor Screening in Computer Simulation Experiments," Technical Report, Georgia Institute of Technology.

Plackett, R. L., and Burman, J. P. (1946). "The Design of Optimum Multifactor Experiments," Biometrika, Vol. 33, p. 305.

Satterthwaite, F. E. (1959). "Random Balance Experimentation," Techometrics, Vol. 1, p. 111.

Smith, D. E., and Mauro, C. A. (1980). "Factor Screening in Computer Simulation: Considerations in Performance Evaluation," Technical Report No. 113-2, Desmatics, Inc.

Watson, G. S. (1961). "A Study of the Group Screening Method," Techometrics, Vol. 3, p. 371.

Youden, W. J., Kempthorne, O., Tukey, J. W., Box, G. E. P., and Hunter, J. S. (1959). "Discussion of the Papers of Messrs. Satterthwaite and Budne," Techometrics, Vol. 1, p. 157.

APPENDIX

The following pages contain the results of the calculations for the twelve specific cases of (K,k) examined. Corresponding to each value of c (c = .1, .2, ..., .9) and α (α = .05, .10, ..., .70) is a block of four numbers. The numbers within a block are arranged as follows:

E(A), the expected number of active factors correctly identified,

E(R), the expected total number of runs,

100 E(F), where E(F) is the classification efficiency for active factor detection,

and 100 E(Q), where E(Q) is the relative testing cost.

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.08 17.64 54.10 27.57	1.08 17.64 54.10 27.57	1.08 17.64 54.10 27.57	1.08 17.64 54.10 27.57	2.00 33.93 100.00 53.02	2.00 33.93 100.00 53.02	2.00 33.93 100.00 53.02	2.00 33.93 100.00 53.02	2.00 43.15 100.00 67.43	2.00 43.15 100.00 67.43	2.00 45.47 100.00 71.05	2.00 45.47 100.00 71.05	2.00 45.47 100.00 71.05	2.00 45.47 100.00 71.05
c = .2	1.91 20.82 95.50 32.53	1.91 21.17 95.50 33.07	1.91 25.81 95.50 40.32	2.00 30.42 100.00 47.53	2.00 31.23 100.00 48.80	2.00 31.23 100.00 48.80	2.00 31.23 100.00 48.80	2.00 38.13 100.00 59.58	2.00 38.13 100.00 59.58	2.00 48.63 100.00 75.99	2.00 54.72 100.00 85.50	2.00 56.11 100.00 87.68	2.00 56.11 100.00 87.68	2.00 56.11 100.00 87.68
c = .3	2.00 25.05 100.00 39.14	2.00 27.84 100.00 43.49	2.00 29.58 100.00 46.21	2.00 33.93 100.00 53.01	2.00 38.10 100.00 59.53	2.00 39.73 100.00 62.07	2.00 40.94 100.00 63.97	2.00 40.94 100.00 63.97	2.00 44.95 100.00 70.23	2.00 44.95 100.00 70.23	2.00 54.05 100.00 84.46	2.00 64.67 100.00 101.04	2.00 65.01 100.00 101.58	2.00 65.01 100.00 101.58
c = .4	2.00 31.98 100.00 49.97	2.00 35.63 100.00 55.77	2.00 37.78 100.00 59.03	2.00 38.94 100.00 60.84	2.00 41.43 100.00 64.74	2.00 45.44 100.00 70.99	2.00 50.19 100.00 78.42	2.00 50.42 100.00 78.79	2.00 50.71 100.00 79.24	2.00 50.71 100.00 79.24	2.00 58.78 100.00 91.84	2.00 65.39 100.00 102.17	2.00 73.10 100.00 114.22	2.00 73.74 100.00 115.22
c = .5	2.00 37.40 99.95 58.44	2.00 40.30 100.00 62.97	2.00 43.20 100.00 67.50	2.00 46.10 100.00 72.03	2.00 49.00 100.00 76.56	2.00 51.90 100.00 81.09	2.00 54.80 100.00 85.62	2.00 57.70 100.00 90.16	2.00 60.60 100.00 94.69	2.00 63.50 100.00 99.22	2.00 66.40 100.00 103.75	2.00 69.30 100.00 108.28	2.00 72.20 100.00 112.81	2.00 75.10 100.00 117.34
c = .6	2.00 43.40 100.00 67.81	2.00 46.30 100.00 72.34	2.00 49.20 100.00 76.87	2.00 52.10 100.00 81.41	2.00 55.00 100.00 85.94	2.00 57.90 100.00 90.47	2.00 60.80 100.00 95.00	2.00 63.70 100.00 99.53	2.00 66.60 100.00 104.06	2.00 69.50 100.00 108.59	2.00 72.40 100.00 113.12	2.00 75.30 100.00 117.66	2.00 78.20 100.00 122.19	2.00 81.10 100.00 126.72
c = .7	2.00 49.40 100.00 77.19	2.00 52.30 100.00 81.72	2.00 55.20 100.00 86.25	2.00 58.10 100.00 90.78	2.00 61.00 100.00 95.31	2.00 63.90 100.00 99.84	2.00 66.80 100.00 104.37	2.00 69.70 100.00 108.91	2.00 72.60 100.00 113.44	2.00 75.50 100.00 117.97	2.00 78.40 100.00 122.50	2.00 81.30 100.00 127.03	2.00 84.20 100.00 131.56	2.00 87.10 100.00 136.09
c = .8	2.00 55.40 100.00 86.56	2.00 58.30 100.00 91.09	2.00 61.20 100.00 95.62	2.00 64.10 100.00 100.16	2.00 67.00 100.00 104.69	2.00 69.90 100.00 109.22	2.00 72.80 100.00 113.75	2.00 75.70 100.00 118.28	2.00 78.60 100.00 122.81	2.00 81.50 100.00 127.34	2.00 84.40 100.00 131.87	2.00 87.30 100.00 136.41	2.00 90.20 100.00 140.94	2.00 93.10 100.00 145.47
c = .9	2.00 61.40 100.00 95.94	2.00 64.30 100.00 100.47	2.00 67.20 100.00 105.00	2.00 70.10 100.00 109.53	2.00 73.00 100.00 114.06	2.00 75.90 100.00 118.59	2.00 78.80 100.00 121.12	2.00 81.70 100.00 127.66	2.00 84.60 100.00 132.19	2.00 87.50 100.00 136.72	2.00 90.40 100.00 141.25	2.00 93.30 100.00 145.78	2.00 96.20 100.00 150.31	2.00 99.10 100.00 154.84

Table 1: Performance Measures for K = 60 and k = 2. (Entries corresponding to values of c greater than 4 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	0.70	0.96	0.96	1.59	1.59	2.10	2.10	2.10	2.10	2.10	2.68	2.68	2.68	2.95
	12.56	14.59	14.59	21.77	21.77	29.64	29.64	29.98	31.52	31.52	45.15	45.15	45.15	54.88
	23.27	32.17	32.17	52.87	52.87	70.00	70.00	70.00	70.00	70.00	89.23	89.23	89.23	98.33
	19.63	22.80	22.80	34.02	34.02	46.31	46.31	46.84	49.25	49.25	70.55	70.55	70.55	85.76
c = .2	1.78	2.16	2.18	2.48	2.63	2.73	2.73	2.73	2.84	2.92	2.93	2.93	2.93	2.93
	20.44	22.99	23.58	26.95	30.24	35.12	37.35	37.57	41.17	46.38	51.69	56.08	59.32	59.67
	59.33	72.17	72.80	82.67	87.70	90.87	90.87	90.87	94.80	97.47	97.67	97.67	97.67	97.67
	31.94	35.92	36.85	42.12	47.25	54.88	58.35	58.71	64.33	72.47	80.76	87.62	92.70	93.23
c = .3	2.23	2.70	2.81	2.87	2.88	2.91	2.95	2.97	2.98	2.98	2.98	2.98	2.99	2.98
	25.23	30.32	32.20	34.94	38.82	40.96	43.11	46.05	49.98	55.46	57.11	57.51	57.51	57.68
	74.20	89.97	93.73	95.70	95.87	96.97	98.30	99.13	99.27	99.30	99.30	99.30	99.30	99.43
	39.43	47.38	50.32	54.60	60.66	64.01	67.37	71.95	78.10	86.65	89.23	89.86	89.86	90.13
c = .4	2.74	2.85	2.93	2.94	2.95	2.97	2.99	2.99	2.99	2.99	2.99	3.00	3.00	3.00
	32.54	35.39	38.66	41.86	43.36	44.46	49.04	53.20	55.59	56.45	57.31	58.85	65.92	75.21
	91.20	95.10	97.60	97.93	98.43	99.00	99.53	99.53	99.57	99.57	99.83	99.90	99.93	99.93
	50.85	55.30	60.41	65.41	67.75	69.47	76.62	83.12	86.86	88.20	89.55	91.96	103.00	117.52
c = .5	2.90	2.96	2.98	2.99	2.99	2.99	2.99	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	37.74	41.39	44.77	48.08	50.60	54.47	56.98	59.38	59.84	62.29	67.88	73.52	74.09	74.09
	96.83	98.70	99.43	99.53	99.77	99.80	99.80	99.93	99.97	100.00	100.00	100.00	100.00	100.00
	58.97	64.67	69.96	75.13	79.06	85.12	89.04	92.78	93.50	97.33	106.06	114.87	115.76	115.76
c = .6	2.95	2.98	2.99	2.99	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	44.30	47.18	50.04	52.89	55.75	58.60	61.45	64.30	67.15	70.00	72.85	75.70	78.55	81.40
	98.47	99.40	99.70	99.83	99.90	99.93	99.93	99.97	99.97	99.97	100.00	100.00	100.00	100.00
	69.22	73.72	78.19	82.65	87.10	91.56	96.01	100.47	104.92	109.37	113.83	118.28	122.73	127.19
c = .7	2.98	2.99	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	50.33	53.19	56.05	58.90	61.75	64.60	67.45	70.30	73.15	76.00	78.85	81.70	84.55	87.40
	99.40	99.80	99.90	99.93	99.97	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	78.64	83.12	87.57	92.03	96.48	100.94	105.39	109.84	114.30	118.75	123.20	127.66	132.11	136.56
c = .8	2.99	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	56.34	59.20	62.05	64.90	67.75	70.60	73.45	76.30	79.15	82.00	84.85	87.70	90.55	93.40
	99.77	99.93	99.97	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	88.04	92.50	96.95	101.41	105.86	110.31	114.77	119.22	123.67	128.12	132.58	137.03	141.48	145.94
c = .9	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00
	62.35	65.20	68.05	70.90	73.75	76.60	79.45	82.30	85.15	88.00	90.85	93.70	96.55	99.40
	99.90	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	97.42	101.87	106.33	110.78	115.23	119.69	124.14	128.59	133.05	137.50	141.95	146.41	150.86	155.31

Table 2: Performance Measures for K = 60 and k = 3. (Entries corresponding to values of c greater than 5 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	0.71	1.15	1.23	1.96	2.09	2.84	2.87	2.97	3.27	3.37	3.79	3.79	3.79	4.17
	11.91	14.27	14.51	20.20	21.20	29.32	29.68	30.99	35.42	37.56	43.37	43.81	43.81	49.63
	14.24	22.92	24.58	39.26	41.76	56.78	57.38	59.40	65.38	67.48	75.84	75.88	75.88	83.36
	18.60	22.29	22.68	31.56	33.13	45.82	46.38	48.43	55.34	58.69	67.76	68.45	68.45	77.54
c = .2	1.64	2.26	2.76	3.03	3.35	3.57	3.88	3.96	4.10	4.22	4.36	4.50	4.67	4.73
	18.79	21.71	25.02	27.27	31.05	33.91	38.13	39.91	42.86	45.72	48.67	51.94	58.22	62.35
	32.86	45.26	55.28	60.66	67.02	71.48	77.68	79.26	82.04	84.32	87.20	89.92	93.44	94.70
	29.35	33.93	39.09	42.61	48.52	52.99	59.58	62.36	66.96	71.44	76.05	81.16	90.97	97.43
c = .3	2.48	3.14	3.51	3.81	4.04	4.21	4.32	4.38	4.43	4.54	4.65	4.71	4.73	4.73
	25.18	28.65	31.77	34.33	37.25	40.05	41.99	44.50	46.68	49.97	53.51	58.64	61.22	61.53
	49.54	62.84	70.20	76.22	80.82	84.22	86.38	87.60	88.54	90.78	93.02	94.26	94.56	94.62
	39.34	44.77	49.64	53.65	58.21	62.58	65.61	69.53	72.94	78.08	83.60	91.63	95.66	96.14
c = .4	3.05	3.66	4.03	4.27	4.38	4.52	4.61	4.68	4.74	4.79	4.81	4.82	4.84	4.92
	32.36	35.72	39.09	41.95	44.33	47.70	51.15	54.16	55.68	59.46	63.15	64.20	64.77	69.56
	61.06	73.20	80.64	85.42	87.62	90.38	92.12	93.62	94.72	95.74	96.28	96.38	96.86	98.44
	50.57	55.81	61.07	65.55	69.26	74.53	79.92	84.62	87.00	92.90	98.68	100.31	101.20	108.68
c = .5	3.62	4.16	4.42	4.59	4.72	4.77	4.80	4.86	4.88	4.89	4.91	4.96	4.97	4.97
	39.23	42.90	45.84	48.82	51.88	54.56	56.25	59.05	62.68	64.83	66.38	69.18	74.75	78.54
	72.32	83.24	88.38	91.74	94.34	95.40	96.08	97.14	97.64	97.82	98.26	99.20	99.48	99.48
	61.30	67.02	71.63	76.28	81.07	85.26	87.89	92.27	97.94	101.30	103.72	108.09	116.80	122.72
c = .6	4.15	4.51	4.68	4.77	4.83	4.87	4.90	4.92	4.94	4.95	4.96	4.97	4.97	4.98
	45.40	48.51	51.42	54.27	57.08	59.87	62.65	65.42	68.19	70.95	73.71	76.47	79.22	81.98
	83.00	97.22	93.50	95.36	96.56	97.38	97.96	98.40	98.72	98.98	99.18	99.34	99.48	99.58
	70.94	75.80	80.35	84.79	89.18	93.55	97.89	102.22	106.54	110.86	115.17	119.48	123.79	128.09
c = .7	4.43	4.69	4.80	4.86	4.90	4.93	4.94	4.96	4.97	4.97	4.98	4.98	4.99	4.99
	51.68	54.69	57.55	60.36	63.15	65.93	68.69	71.46	74.22	76.97	79.73	82.48	85.24	87.99
	88.52	93.82	96.06	97.28	98.04	98.54	98.90	99.14	99.34	99.48	99.58	99.68	99.74	99.80
	80.74	85.45	89.93	94.32	98.68	103.01	107.34	111.65	115.96	120.27	124.58	128.88	133.18	137.48
c = .8	4.62	4.81	4.88	4.92	4.94	4.96	4.97	4.98	4.98	4.99	4.99	4.99	4.99	4.99
	57.87	60.91	63.63	66.42	69.19	71.96	74.72	77.48	80.23	82.99	85.74	88.49	91.24	94.00
	92.38	96.14	97.64	98.42	98.90	99.20	99.40	99.54	99.66	99.74	99.80	99.84	99.88	99.90
	90.42	95.01	99.43	103.78	108.12	112.44	116.75	121.06	125.36	129.67	133.97	138.27	142.57	146.87
c = .9	4.75	4.89	4.93	4.95	4.97	4.98	4.98	4.99	4.99	4.99	4.99	5.00	5.00	5.00
	64.00	66.88	69.68	72.45	75.22	77.98	80.73	83.49	86.24	88.99	91.74	94.50	97.25	100.00
	95.00	97.62	98.60	99.10	99.38	99.56	99.68	99.76	99.82	99.86	99.90	99.92	99.94	99.96
	100.00	104.50	108.87	113.21	117.53	121.84	126.15	130.45	134.75	139.05	143.35	147.65	151.95	156.25

Table 3: Performance Measures for K = 60 and k = 5. (Entries corresponding to values of c greater than 5 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	0.97	1.60	2.08	2.57	3.24	3.62	4.01	4.15	4.64	4.84	5.44	5.73	6.07	6.53
	12.90	15.93	18.69	21.37	26.14	28.97	31.91	32.98	37.48	38.77	43.48	46.37	49.99	54.19
	12.16	20.04	26.01	32.16	40.50	45.24	50.15	51.89	58.05	60.49	68.00	71.65	75.92	81.65
	20.16	24.89	29.21	33.39	40.85	45.26	49.86	51.54	58.56	60.57	67.93	72.45	78.10	84.67
c = .2	1.69	2.52	3.16	3.72	4.26	4.68	4.99	5.36	5.64	5.92	6.11	6.26	6.47	6.84
	18.95	22.79	26.40	29.66	33.06	35.66	37.69	40.92	43.70	46.79	49.78	51.64	53.73	57.53
	21.15	31.52	39.51	46.54	53.26	58.49	62.32	66.97	70.55	74.04	76.37	78.19	80.82	85.46
	29.61	35.62	41.25	46.35	51.66	55.72	58.88	63.93	68.28	73.10	77.78	80.69	83.95	89.89
c = .3	2.46	3.54	4.29	4.75	5.24	5.54	5.96	6.18	6.42	6.64	6.86	6.99	7.07	7.16
	25.46	29.29	32.69	35.49	38.58	41.17	44.97	47.74	50.42	53.20	56.38	58.85	60.44	61.87
	30.80	44.27	53.60	59.38	65.51	69.20	74.46	77.25	80.20	83.05	85.76	87.40	88.40	89.45
	39.78	45.77	51.08	55.46	60.29	64.32	70.26	74.59	78.78	83.12	88.09	91.95	94.43	96.67
c = .4	3.25	4.39	5.04	5.59	5.92	6.24	6.48	6.75	6.89	7.01	7.20	7.35	7.41	7.44
	32.14	36.14	38.92	42.49	45.37	48.71	51.39	54.52	57.21	59.67	62.25	66.03	68.64	69.50
	40.60	54.90	62.99	69.82	73.99	78.04	81.01	84.41	86.16	87.66	90.04	91.82	92.60	92.96
	50.22	56.47	60.82	66.38	70.89	76.10	80.30	85.19	89.39	93.23	97.27	103.18	107.26	108.60
c = .5	4.01	5.05	5.72	6.11	6.45	6.69	6.91	7.02	7.19	7.31	7.39	7.52	7.62	7.64
	39.84	43.47	47.37	50.78	53.91	56.79	60.19	62.19	65.20	67.80	69.98	72.37	75.92	77.29
	50.09	63.16	71.44	76.39	80.59	83.60	86.37	87.80	89.84	91.39	92.31	94.04	95.27	95.55
	62.26	67.93	74.01	79.34	84.24	88.74	94.04	97.17	101.88	105.94	109.34	113.09	118.62	120.76
c = .6	4.77	5.75	6.29	6.65	6.91	7.10	7.26	7.38	7.48	7.56	7.63	7.69	7.74	7.79
	45.87	49.45	52.59	55.55	58.41	61.20	63.96	66.68	69.38	72.06	74.73	77.39	80.05	82.69
	59.63	71.82	78.64	83.14	86.36	88.80	90.70	92.24	93.49	94.52	95.41	96.16	96.81	97.40
	71.67	77.26	82.17	86.80	91.26	95.63	99.93	104.19	108.40	112.60	116.77	120.93	125.07	129.21
c = .7	5.33	6.22	6.69	6.99	7.20	7.35	7.47	7.56	7.64	7.70	7.75	7.79	7.83	7.86
	52.43	55.92	58.99	61.89	64.70	67.45	70.17	72.86	75.54	78.20	80.85	83.49	86.13	88.76
	66.65	77.72	83.60	87.35	89.96	91.90	93.38	94.54	95.47	96.24	96.88	97.41	97.87	98.27
	81.93	87.37	92.17	96.70	101.09	105.39	109.64	113.85	118.03	122.19	126.33	130.46	134.58	138.69
c = .8	5.82	6.60	7.00	7.24	7.41	7.53	7.62	7.69	7.75	7.79	7.83	7.86	7.89	7.91
	58.92	62.30	65.30	68.14	70.91	73.63	76.32	78.99	81.65	84.29	86.93	89.56	92.19	94.81
	72.70	82.51	87.49	90.55	92.65	94.15	95.29	96.16	96.86	97.41	97.87	98.25	98.57	98.85
	92.06	97.34	102.03	106.48	110.80	115.05	119.25	123.43	127.58	131.71	135.83	139.94	144.04	148.14
c = .9	6.23	6.91	7.24	7.44	7.57	7.66	7.73	7.78	7.83	7.86	7.88	7.91	7.92	7.94
	65.33	68.61	71.54	74.34	77.07	79.76	82.43	85.08	87.73	90.36	92.98	95.61	98.22	100.84
	77.81	86.35	90.49	92.97	94.62	95.80	96.66	97.31	97.82	98.22	98.55	98.82	99.05	99.24
	102.07	107.20	111.78	116.15	120.42	124.63	128.80	132.95	137.07	141.18	145.29	149.38	153.47	157.56

Table 4: Performance Measures for K = 60 and k = 8. (Entries corresponding to values of c greater than 5 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.64	2.25	2.52	2.86	3.07	3.26	3.34	3.43	3.43	3.53	3.68	3.77	3.85	3.86
	23.45	29.39	34.18	42.88	48.89	54.54	58.43	65.02	66.18	71.74	79.19	88.34	101.29	109.30
	41.00	56.17	62.88	71.42	76.77	81.60	83.38	85.63	85.63	88.37	91.97	94.37	96.20	96.42
	18.91	23.70	27.57	34.58	39.43	43.98	47.12	52.44	53.37	57.86	63.87	71.24	81.68	88.15
c = .2	3.02	3.41	3.64	3.74	3.81	3.83	3.88	3.93	3.94	3.94	3.95	3.95	3.97	3.98
	36.13	40.93	47.54	54.14	58.73	65.47	70.98	76.83	84.50	92.39	96.68	97.39	100.53	116.32
	75.50	85.37	91.07	93.60	95.15	95.67	96.90	98.15	98.55	98.62	98.72	98.85	99.17	99.62
	29.14	33.01	38.34	43.66	47.36	52.80	57.24	61.96	68.14	74.51	77.97	78.54	81.07	93.81
c = .3	3.63	3.80	3.88	3.91	3.93	3.96	3.97	3.97	3.98	3.99	3.99	3.99	3.99	3.99
	46.77	52.62	59.32	63.87	68.65	73.66	80.98	86.78	89.81	95.27	104.89	110.35	110.70	113.48
	90.65	95.02	97.05	97.85	98.35	98.95	99.13	99.25	99.55	99.65	99.67	99.67	99.70	99.82
	37.71	42.44	47.84	51.51	55.36	59.40	65.30	69.99	72.43	76.83	84.59	88.99	89.27	91.52
c = .4	3.90	3.96	3.98	3.99	3.99	3.99	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	60.90	65.71	70.26	75.95	83.26	89.06	92.54	100.43	107.51	109.13	114.47	125.84	130.01	130.13
	97.52	99.02	99.50	99.67	99.77	99.85	99.90	99.90	99.90	99.90	99.92	99.92	99.92	99.92
	49.11	52.99	56.66	61.25	67.14	71.82	74.63	80.99	86.70	88.01	92.31	101.48	104.85	104.94
c = .5	3.97	3.99	3.99	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	72.27	78.09	83.89	89.70	95.50	101.30	107.10	112.90	118.70	124.50	130.30	136.10	141.90	147.70
	99.25	99.72	99.85	99.92	99.95	99.97	99.97	99.97	100.00	100.00	100.00	100.00	100.00	100.00
	58.28	62.97	67.66	72.34	77.01	81.69	86.37	91.05	95.73	100.40	105.08	109.76	114.44	119.11
c = .6	3.99	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	84.29	90.10	95.90	101.70	107.50	113.30	119.10	124.90	130.70	136.50	142.30	148.10	153.90	159.70
	99.80	99.92	99.97	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	67.98	72.66	77.34	82.02	86.69	91.37	96.05	100.73	105.40	110.08	114.76	119.44	124.11	128.79
c = .7	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	96.30	102.10	107.90	113.70	119.50	125.30	131.10	136.90	142.70	148.50	154.30	160.10	165.90	171.70
	99.95	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	77.66	82.34	87.02	91.69	96.37	101.05	105.73	110.40	115.08	119.76	124.44	129.11	133.79	138.47
c = .8	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	108.30	114.10	119.90	125.70	131.50	137.30	143.10	148.90	154.70	160.50	166.30	172.10	177.90	183.70
	99.97	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	87.34	92.02	96.69	101.37	106.05	110.73	115.40	120.08	124.76	129.44	134.11	138.79	143.47	148.15
c = .9	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00
	120.30	126.10	131.90	137.70	143.50	149.30	155.10	160.90	166.70	172.50	178.30	184.10	189.90	195.70
	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	97.02	101.69	106.37	111.05	115.73	120.40	125.08	129.76	134.44	139.11	143.79	148.47	153.15	157.82

Table 5: Performance Measures for K = 120 and k = 4. (Entries corresponding to values of c greater than 4 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.69	2.37	2.88	3.37	3.70	4.02	4.20	4.47	4.64	4.77	4.87	5.01	5.29	5.51
	22.80	28.73	35.05	41.13	47.84	54.21	58.60	66.17	70.09	75.93	79.90	83.69	93.89	104.82
	28.18	39.50	48.05	56.20	61.75	67.07	70.00	74.52	77.27	79.53	81.10	83.48	88.18	91.77
	18.39	23.17	28.27	33.17	38.58	43.71	47.26	53.36	56.53	61.23	64.44	67.49	75.72	84.53
c = .2	3.15	3.97	4.43	4.77	4.97	5.21	5.34	5.43	5.49	5.58	5.69	5.71	5.72	5.78
	35.83	41.08	46.37	53.69	59.00	64.34	71.71	76.37	79.66	86.33	96.38	102.04	102.98	108.69
	52.50	66.20	73.75	79.50	82.87	86.80	88.92	90.48	91.58	93.02	94.92	95.23	95.33	96.40
	28.89	33.13	37.39	43.30	47.58	51.89	57.83	61.59	64.24	69.62	77.72	82.29	83.05	87.65
c = .3	4.34	4.92	5.27	5.44	5.56	5.67	5.73	5.78	5.81	5.83	5.86	5.90	5.91	5.91
	49.45	55.28	61.56	67.43	71.99	76.55	81.97	86.46	95.61	100.30	103.64	111.66	120.22	121.59
	72.37	82.05	87.88	90.72	92.70	94.53	95.58	96.42	96.90	97.17	97.68	98.33	98.48	98.48
	39.88	44.58	49.64	54.38	58.06	61.73	66.10	69.73	77.11	80.89	83.58	90.05	96.95	98.05
c = .4	5.09	5.50	5.67	5.77	5.84	5.87	5.90	5.92	5.94	5.95	5.96	5.97	5.98	5.98
	61.80	66.89	72.71	79.15	84.32	89.73	95.55	101.36	105.93	113.24	117.80	120.01	129.68	137.12
	84.87	91.62	94.50	96.13	97.27	97.78	98.33	98.65	99.00	99.20	99.38	99.43	99.65	99.70
	49.83	53.94	58.64	63.83	68.00	72.36	77.06	81.74	85.42	91.33	95.00	96.79	104.58	110.58
c = .5	5.56	5.77	5.86	5.91	5.94	5.95	5.96	5.97	5.98	5.98	5.99	5.99	5.99	5.99
	73.76	79.67	85.46	91.21	96.93	102.65	108.36	114.07	119.78	125.48	131.19	136.89	142.59	148.29
	92.60	96.23	97.68	98.45	98.92	99.20	99.40	99.55	99.65	99.73	99.78	99.83	99.87	99.90
	59.48	64.25	68.92	73.55	78.17	82.78	87.39	91.99	96.60	101.20	105.80	110.40	114.99	119.59
c = .6	5.78	5.90	5.94	5.96	5.97	5.98	5.99	5.99	5.99	5.99	6.00	6.00	6.00	6.00
	85.98	91.80	97.54	103.26	108.97	114.68	120.39	126.09	131.79	137.49	143.20	148.90	154.60	160.30
	96.27	98.27	99.00	99.37	99.57	99.70	99.78	99.83	99.88	99.90	99.93	99.95	99.97	99.97
	69.34	74.03	78.66	83.28	87.88	92.49	97.09	101.69	106.28	110.88	115.48	120.08	124.68	129.27
c = .7	5.89	5.95	5.98	5.98	5.99	5.99	5.99	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	98.09	103.85	109.57	115.28	120.99	126.69	132.40	138.10	143.80	149.50	155.20	160.90	166.60	172.30
	98.17	99.22	99.58	99.73	99.83	99.88	99.92	99.95	99.95	99.97	99.98	99.98	99.98	99.98
	79.11	83.75	88.37	92.97	97.57	102.17	106.77	111.37	115.97	120.56	125.16	129.76	134.35	138.95
c = .8	5.95	5.98	5.99	5.99	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	110.15	115.88	121.59	127.29	133.00	138.70	144.40	150.10	155.80	161.50	167.20	172.90	178.60	184.30
	99.13	99.67	99.82	99.90	99.93	99.95	99.97	99.98	99.98	99.98	100.00	100.00	100.00	100.00
	88.83	93.45	98.06	102.66	107.25	111.85	116.45	121.05	125.64	130.24	134.84	139.44	144.03	148.63
c = .9	5.98	5.99	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
	122.18	127.89	133.60	139.30	145.00	150.70	156.40	162.10	167.80	173.50	179.20	184.90	190.60	196.30
	99.58	99.85	99.93	99.97	99.98	99.98	99.98	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	98.53	103.14	107.74	112.34	116.93	121.53	126.13	130.73	135.32	139.92	144.52	149.11	153.71	158.31

Table 6: Performance Measures for K = 120 and k = 6. (Entries corresponding to values of c greater than 4 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.72	2.66	3.49	4.16	4.80	5.34	5.75	6.20	6.57	7.00	7.35	7.57	7.78	8.18
	23.29	30.36	37.00	42.55	48.38	53.47	60.32	67.06	73.71	80.90	86.59	90.74	93.15	100.15
	17.16	26.61	34.89	41.63	48.04	53.41	57.53	61.99	65.66	70.03	73.49	75.66	77.76	81.79
	18.78	24.49	29.84	34.31	39.01	43.12	48.65	54.08	59.44	65.24	69.83	73.18	75.12	80.77
c = .2	3.24	4.56	5.38	6.15	6.64	7.15	7.49	7.81	8.08	8.37	8.56	8.80	8.98	9.07
	33.67	40.80	46.02	52.14	58.92	66.65	73.27	78.47	83.45	90.66	95.24	102.08	106.83	110.84
	32.40	45.60	53.81	61.50	66.44	71.51	74.86	78.10	80.81	83.71	85.59	87.97	89.84	90.70
	27.15	32.91	37.12	42.04	47.51	53.75	59.09	63.28	67.30	73.11	76.81	82.32	86.15	89.39
c = .3	4.87	6.21	6.99	7.52	7.93	8.26	8.50	8.72	8.90	9.04	9.21	9.31	9.42	9.57
	49.07	56.72	63.81	69.44	76.06	82.59	87.34	92.46	97.65	102.59	108.49	114.36	118.32	126.99
	48.67	62.06	69.92	75.21	79.32	82.58	85.01	87.15	89.05	90.40	92.12	93.15	94.20	95.70
	39.57	45.74	51.46	56.00	61.34	66.61	70.43	74.56	78.75	82.73	87.49	92.23	95.42	102.42
c = .4	6.04	7.24	7.90	8.39	8.70	8.97	9.13	9.28	9.40	9.50	9.54	9.65	9.69	9.76
	62.65	69.84	74.65	80.28	85.11	91.62	96.05	101.34	105.98	113.04	117.24	123.13	128.94	134.78
	60.41	72.39	78.97	83.87	87.02	89.72	91.30	92.83	94.03	94.99	95.42	96.52	96.92	97.55
	50.53	56.32	60.20	64.74	68.64	73.88	77.46	81.72	85.47	91.16	94.55	99.30	103.99	108.69
c = .5	7.08	8.13	8.65	8.97	9.18	9.34	9.46	9.56	9.64	9.70	9.76	9.78	9.83	9.83
	75.72	82.28	89.46	95.66	100.45	104.93	111.07	115.48	122.10	127.92	134.68	138.00	144.10	147.13
	70.77	81.34	86.46	89.67	91.75	93.38	94.64	95.60	96.37	96.98	97.60	97.80	98.27	98.32
	61.07	66.35	72.14	77.15	81.01	84.62	89.57	93.13	98.47	103.16	108.62	111.29	116.21	118.66
c = .6	7.86	8.69	9.09	9.35	9.48	9.60	9.71	9.78	9.81	9.85	9.88	9.89	9.92	9.93
	86.48	93.64	98.44	103.37	109.56	116.00	120.79	127.46	132.16	139.35	144.60	151.22	155.64	162.80
	78.57	86.92	90.92	93.50	94.85	96.00	97.12	97.82	98.13	98.52	98.77	98.92	99.17	99.27
	69.74	75.52	79.39	83.37	88.35	93.55	97.41	102.79	106.58	112.38	116.61	121.95	125.52	131.29
c = .7	8.52	9.13	9.42	9.60	9.70	9.77	9.84	9.87	9.89	9.92	9.94	9.95	9.96	9.97
	100.74	107.29	111.76	116.38	126.56	132.57	136.82	141.69	145.45	152.30	158.04	164.65	167.96	176.55
	85.18	91.30	94.24	96.04	96.98	97.74	98.42	98.74	98.94	99.16	99.38	99.48	99.56	99.68
	81.24	86.52	90.13	93.83	102.06	106.91	110.34	114.27	117.30	122.82	127.45	132.78	135.45	142.38
c = .8	8.98	9.48	9.67	9.78	9.85	9.89	9.92	9.94	9.95	9.95	9.97	9.97	9.97	9.98
	112.43	119.80	123.57	129.73	134.47	140.57	145.82	152.72	157.12	161.25	167.86	174.47	181.07	187.12
	89.85	94.80	96.67	97.85	98.47	98.92	99.17	99.42	99.50	99.55	99.65	99.70	99.70	99.75
	90.67	96.62	99.65	104.62	108.45	113.36	117.59	123.16	126.71	130.04	135.37	140.70	146.02	150.91
c = .9	9.27	9.61	9.78	9.84	9.88	9.91	9.94	9.95	9.97	9.98	9.98	9.99	9.99	9.99
	124.17	129.54	136.31	142.34	147.72	155.29	160.67	166.97	172.32	179.56	185.85	191.51	196.86	199.69
	92.71	96.11	97.77	98.43	98.80	99.09	99.37	99.54	99.66	99.77	99.80	99.86	99.86	99.86
	100.14	104.47	109.92	114.79	119.13	125.24	129.57	134.65	138.97	144.81	149.88	154.45	158.76	161.04

Table 7: Performance Measures for K = 120 and k = 10. (All entries are Monte Carlo estimates.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.99	3.24	4.46	5.50	6.45	7.20	8.06	8.72	9.43	10.17	10.86	11.47	12.14	12.49
	20.23	26.27	31.64	40.39	48.20	53.11	61.45	67.11	72.60	77.71	83.81	88.79	95.90	100.00
	12.42	20.26	27.85	34.40	40.31	44.99	50.36	54.49	58.91	63.55	67.87	71.71	75.86	78.09
	16.32	21.19	25.52	32.57	38.87	42.83	49.56	54.12	58.55	62.67	67.59	71.60	77.34	80.65
c = .2	3.51	5.39	6.65	7.58	8.49	9.29	10.15	10.73	11.25	11.83	12.38	12.82	13.22	13.79
	34.38	41.67	47.71	54.25	60.78	68.23	74.29	80.29	85.39	90.95	98.16	102.55	107.53	115.38
	21.96	33.70	41.55	47.35	53.06	58.04	63.41	67.06	70.29	73.94	77.40	80.11	82.61	86.19
	27.73	33.60	38.47	43.75	49.02	55.03	59.91	64.75	68.85	73.35	79.16	82.70	86.71	93.05
c = .3	5.13	7.07	8.46	9.57	10.40	11.15	11.75	12.28	12.73	13.16	13.50	13.79	14.15	14.35
	49.65	56.68	62.95	72.16	77.86	82.77	87.08	92.71	97.55	104.24	110.37	114.13	119.83	123.03
	32.06	44.17	52.89	59.84	65.02	69.66	73.42	76.77	79.54	82.24	84.38	86.17	88.46	89.67
	40.04	45.71	50.76	58.20	62.79	66.75	70.22	74.76	78.67	84.06	89.01	92.04	96.63	99.22
c = .4	6.59	8.75	10.07	11.05	11.80	12.34	12.80	13.21	13.53	13.84	14.11	14.46	14.68	14.92
	63.12	71.52	79.08	85.47	92.88	99.03	104.90	109.68	117.69	124.24	128.46	133.19	137.14	143.01
	41.20	54.69	62.94	69.06	73.75	77.14	80.01	82.55	84.54	86.49	88.17	90.40	91.72	93.27
	50.91	57.68	63.78	68.93	74.90	79.87	84.60	88.45	94.91	100.20	103.60	107.41	110.60	115.33
c = .5	8.05	10.08	11.31	12.16	12.80	13.30	13.71	14.04	14.33	14.57	14.78	14.97	15.13	15.28
	75.74	82.98	89.41	95.46	101.30	107.00	112.61	118.14	123.63	129.07	134.48	139.87	145.23	150.58
	50.28	63.01	70.66	75.99	79.99	83.13	85.66	87.77	89.54	91.06	92.39	93.55	94.59	95.52
	61.08	66.92	72.10	76.98	81.69	86.29	90.81	95.28	99.70	104.09	108.45	112.80	117.12	121.44
c = .6	9.27	11.20	12.30	13.04	13.58	14.00	14.32	14.59	14.81	14.99	15.15	15.28	15.40	15.51
	88.97	96.10	102.40	108.34	114.08	119.70	125.22	130.69	136.11	141.49	146.85	152.18	157.50	162.81
	57.96	70.02	76.90	81.52	84.89	87.47	89.52	91.17	92.54	93.69	94.68	95.53	96.28	96.94
	71.75	77.50	82.58	87.37	92.00	96.53	100.99	105.39	109.76	114.11	118.43	122.73	127.02	131.30
c = .7	10.36	12.14	13.11	13.74	14.18	14.52	14.78	14.98	15.15	15.29	15.40	15.51	15.59	15.67
	102.06	109.04	115.21	121.04	126.68	132.22	137.68	143.08	148.45	153.79	159.11	164.40	169.69	174.97
	64.76	75.87	81.92	85.85	88.64	90.73	92.35	93.64	94.69	95.56	96.28	96.91	97.44	97.91
	82.31	87.93	92.91	97.61	102.16	106.63	111.03	115.39	119.72	124.02	128.31	132.58	136.85	141.10
c = .8	11.31	12.91	13.75	14.27	14.64	14.90	15.11	15.27	15.40	15.50	15.59	15.66	15.72	15.77
	115.01	121.81	127.85	133.57	139.14	144.61	150.01	155.37	160.70	166.00	171.29	176.56	181.82	187.07
	70.67	80.69	85.91	89.21	91.49	93.16	94.42	95.42	96.22	96.87	97.41	97.96	98.24	98.57
	92.75	98.24	103.10	107.72	112.21	116.62	120.97	125.30	129.59	133.87	138.13	142.38	146.63	150.86
c = .9	12.12	13.54	14.25	14.69	14.98	15.19	15.35	15.47	15.57	15.65	15.71	15.76	15.81	15.84
	127.82	134.44	140.35	145.99	151.48	156.89	162.25	167.57	172.87	178.15	183.41	188.66	193.91	199.14
	75.77	84.64	89.08	91.80	93.64	94.96	95.95	96.71	97.31	97.79	98.19	98.52	98.79	99.02
	103.08	108.42	113.19	117.73	122.16	126.53	130.85	135.14	139.41	143.67	147.91	152.15	156.38	160.60

Table 8: Performance Measures for K = 120 and k = 16. (Entries corresponding to values of c greater than 4 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
	3.27	4.38	4.98	5.54	5.91	6.23	6.47	6.71	6.85	6.97	7.15	7.32	7.39	7.42
c = .1	42.53	55.47	66.05	81.23	92.73	104.88	116.72	133.67	144.71	154.58	167.52	180.44	189.79	194.23
	40.82	54.76	62.24	69.30	73.94	77.86	80.84	83.88	85.59	87.17	89.44	91.49	92.34	92.70
	17.43	22.73	27.07	33.29	38.01	42.98	47.84	54.78	59.31	63.35	68.66	73.95	77.78	79.60
	5.76	6.61	6.99	7.26	7.39	7.53	7.61	7.70	7.76	7.80	7.85	7.87	7.89	7.93
c = .2	66.96	83.53	96.40	110.59	125.00	134.06	146.27	158.86	171.05	182.15	196.13	211.49	216.86	232.60
	71.94	82.69	87.39	90.77	92.42	94.14	95.11	96.31	97.02	97.52	98.19	98.40	98.56	99.07
	27.44	34.23	39.51	45.32	51.23	54.94	59.95	65.11	70.10	74.65	80.38	86.68	88.88	95.33
	7.04	7.46	7.66	7.76	7.82	7.86	7.91	7.94	7.95	7.97	7.97	7.98	7.98	7.99
c = .3	93.83	105.52	116.98	131.93	139.68	154.58	166.40	176.16	186.41	200.77	211.53	217.69	231.01	236.13
	88.02	93.30	95.72	96.94	97.74	98.31	98.86	99.22	99.36	99.61	99.64	99.75	99.81	99.84
	38.46	43.25	47.94	54.07	57.25	63.35	68.20	72.20	76.40	82.28	86.69	89.22	94.67	96.77
	7.65	7.83	7.90	7.94	7.96	7.97	7.98	7.98	7.99	7.99	7.99	7.99	7.99	8.00
c = .4	117.75	129.53	141.20	152.84	164.46	176.07	187.68	199.28	210.89	222.49	234.09	245.69	257.30	268.90
	95.59	97.89	98.75	99.19	99.45	99.60	99.71	99.79	99.84	99.87	99.90	99.92	99.94	99.96
	48.26	53.09	57.87	62.64	67.40	72.16	76.92	81.67	86.43	91.18	95.94	100.69	105.45	110.20
	7.87	7.95	7.97	7.98	7.99	7.99	7.99	8.00	8.00	8.00	8.00	8.00	8.00	8.00
c = .5	141.97	153.65	165.27	176.88	188.49	200.09	211.69	223.30	234.90	246.50	258.10	269.70	281.30	292.90
	98.41	99.32	99.64	99.77	99.85	99.90	99.92	99.95	99.96	99.97	99.97	99.99	99.99	99.99
	58.19	62.97	67.73	72.49	77.25	82.00	86.76	91.51	96.27	101.02	105.78	110.53	115.29	120.04
	7.96	7.98	7.99	7.99	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
c = .6	166.06	177.68	189.29	200.90	212.50	224.10	235.70	247.30	258.90	270.50	282.10	293.70	305.30	316.90
	99.45	99.80	99.90	99.94	99.96	99.97	99.99	99.99	99.99	100.00	100.00	100.00	100.00	100.00
	68.06	72.82	77.58	82.33	87.09	91.84	96.60	101.35	106.11	110.86	115.61	120.37	125.12	129.88
	7.99	7.99	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
c = .7	190.09	201.70	213.30	224.90	236.50	248.10	259.70	271.30	282.90	294.50	306.10	317.70	329.30	340.90
	99.82	99.94	99.97	99.99	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	77.90	82.66	87.42	92.17	96.93	101.68	106.43	111.19	115.94	120.70	125.45	130.20	134.96	139.71
	7.99	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
c = .8	214.10	225.70	237.30	248.90	260.50	272.10	283.70	295.30	306.90	318.50	330.10	341.70	353.30	364.90
	99.94	99.99	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	87.74	92.50	97.25	102.01	106.76	111.52	116.27	121.02	125.78	130.53	135.29	140.04	144.79	149.55
	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00
c = .9	238.10	249.70	261.30	272.90	284.50	296.10	307.70	319.30	330.90	342.50	354.10	365.70	377.30	388.90
	99.99	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	97.58	102.34	107.09	111.84	116.60	121.35	126.11	130.86	135.61	140.37	145.12	149.88	154.63	159.39

Table 9: Performance Measures for K = 240 and k = 8. (Entries corresponding to values of c greater than 3 are asmptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	3.46	4.93	5.96	6.77	7.39	7.94	8.41	8.81	9.20	9.54	9.87	10.15	10.45	10.73
	43.07	54.32	66.79	80.71	93.89	107.84	121.42	134.94	146.49	158.83	168.93	178.98	192.12	206.07
	28.81	41.09	49.68	56.38	61.56	66.17	70.09	73.44	76.68	79.47	82.23	84.62	87.12	89.45
	17.65	22.26	27.37	33.08	38.48	44.19	49.76	55.30	60.04	65.09	69.23	73.35	78.74	84.46
c = .2	6.23	7.87	8.72	9.36	9.83	10.22	10.51	10.77	10.92	11.10	11.24	11.39	11.53	11.59
	64.73	76.87	89.22	102.86	111.33	124.72	135.01	145.27	160.92	173.10	183.74	194.39	210.03	216.09
	51.88	65.55	72.67	78.02	81.91	85.16	87.59	89.75	90.99	92.49	93.67	94.94	96.09	96.57
	26.53	31.50	36.57	42.16	45.63	51.11	55.33	59.54	65.95	70.94	75.30	79.67	86.08	88.56
c = .3	8.48	9.64	10.27	10.72	11.03	11.21	11.35	11.45	11.56	11.66	11.71	11.75	11.78	11.83
	94.53	109.39	120.84	132.84	144.70	155.70	165.22	175.41	185.63	197.99	214.64	228.39	239.96	253.72
	70.67	80.32	85.55	89.32	91.93	93.43	94.59	95.38	96.33	97.15	97.60	97.92	98.15	98.57
	38.74	44.83	49.53	54.44	59.30	63.81	67.71	71.89	76.09	81.14	87.97	93.60	99.35	103.98
c = .4	9.99	10.81	11.20	11.42	11.56	11.66	11.74	11.79	11.83	11.86	11.89	11.91	11.93	11.94
	119.89	132.11	143.90	155.52	167.06	178.56	190.04	201.49	212.93	224.36	235.79	247.21	258.63	270.04
	83.25	90.12	93.30	95.15	96.36	97.19	97.80	98.25	98.60	98.87	99.09	99.27	99.41	99.53
	49.14	54.14	58.97	63.74	68.47	73.18	77.88	82.58	87.27	91.95	96.64	101.32	106.00	110.67
c = .5	10.87	11.39	11.61	11.73	11.81	11.86	11.89	11.92	11.93	11.95	11.96	11.97	11.98	11.98
	144.77	156.69	168.31	179.83	191.31	202.76	214.19	225.62	237.03	248.45	259.86	271.27	282.67	294.08
	90.59	94.94	96.77	97.77	98.40	98.81	99.09	99.30	99.45	99.57	99.66	99.73	99.79	99.83
	59.33	64.22	68.98	73.70	78.40	83.10	87.78	92.47	97.15	101.82	106.50	111.18	115.85	120.52
c = .6	11.39	11.70	11.82	11.88	11.92	11.94	11.96	11.97	11.98	11.98	11.98	11.99	11.99	11.99
	169.29	181.00	192.52	203.98	215.42	226.84	238.26	249.67	261.07	272.48	283.88	295.29	306.69	318.09
	94.88	97.47	98.48	99.00	99.30	99.50	99.63	99.72	99.79	99.83	99.87	99.90	99.92	99.94
	69.38	74.18	78.90	83.60	88.29	92.97	97.65	102.32	107.00	111.67	116.35	121.02	125.69	130.37
c = .7	11.68	11.85	11.92	11.95	11.96	11.98	11.98	11.99	11.99	11.99	11.99	12.00	12.00	12.00
	193.57	205.15	216.62	228.05	239.46	250.87	262.28	273.69	285.09	296.49	307.89	319.30	330.70	342.10
	97.29	98.77	99.30	99.56	99.70	99.79	99.85	99.89	99.92	99.94	99.95	99.97	99.97	99.98
	79.33	84.08	88.78	93.46	98.14	102.82	107.49	112.17	116.84	121.51	126.19	130.86	135.53	140.20
c = .8	11.83	11.93	11.96	11.98	11.98	11.99	11.99	11.99	12.00	12.00	12.00	12.00	12.00	12.00
	217.73	229.23	240.66	252.08	263.48	274.89	286.29	297.69	309.10	320.50	331.90	343.30	354.70	366.10
	98.60	99.41	99.68	99.81	99.87	99.92	99.94	99.96	99.97	99.97	99.98	99.98	99.99	99.99
	89.23	93.95	98.63	103.31	107.99	112.66	117.33	122.01	126.68	131.35	136.02	140.70	145.37	150.04
c = .9	11.91	11.97	11.98	11.99	11.99	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00
	241.81	253.27	264.68	276.09	287.49	298.90	310.30	321.70	333.10	344.50	355.90	367.30	378.70	390.10
	99.29	99.72	99.86	99.92	99.95	99.97	99.97	99.98	99.99	99.99	99.99	99.99	100.00	100.00
	99.10	103.80	108.48	113.15	117.83	122.50	127.17	131.84	136.52	141.19	145.86	150.53	155.20	159.88

Table 10: Performance Measures for K = 240 and k = 12. (Entries corresponding to values of c greater than 3 are asymptotic values.)

	Significance Level (α)													
	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
	3.62	5.67	7.26	8.51	9.64	10.64	11.61	12.52	13.32	14.06	14.76	15.40	15.97	16.60
c = .1	45.68	55.94	70.07	82.99	93.63	101.97	114.61	126.33	138.79	151.21	162.71	175.46	190.29	199.13
	18.11	28.33	36.30	42.54	48.22	53.17	58.03	62.62	66.58	70.32	73.78	77.02	79.83	83.00
	18.72	22.93	28.72	34.01	38.37	41.79	46.97	51.78	56.88	61.97	66.68	71.91	77.99	81.61
	6.53	9.34	11.01	12.31	13.41	14.24	15.09	15.77	16.31	16.77	17.18	17.55	17.96	18.32
c = .2	68.65	83.08	94.04	105.42	115.04	124.38	137.63	150.71	164.42	178.82	191.63	202.06	211.77	222.20
	32.67	46.69	55.03	61.55	67.06	71.18	75.42	78.85	81.57	83.84	85.91	87.73	89.79	91.62
	28.14	34.05	38.54	43.20	47.15	50.98	56.40	61.77	67.38	73.29	78.54	82.81	86.79	91.07
	9.58	12.00	13.58	14.69	15.53	16.17	16.69	17.13	17.53	17.87	18.14	18.40	18.60	18.86
c = .3	95.40	109.15	124.75	139.88	147.73	159.15	172.62	181.68	193.95	205.07	217.20	227.16	237.07	243.80
	47.89	60.01	67.89	73.45	77.64	80.86	83.47	85.63	87.67	89.35	90.68	92.00	93.01	94.29
	39.10	44.73	51.13	57.33	60.54	65.23	70.74	74.46	79.49	84.04	89.01	93.10	97.16	99.92
	12.09	14.43	15.74	16.62	17.25	17.73	18.11	18.41	18.66	18.87	19.05	19.21	19.34	19.46
c = .4	121.59	134.93	147.24	159.12	170.75	182.23	193.61	204.91	216.16	227.37	238.55	249.70	260.84	271.96
	60.44	72.13	78.70	83.08	86.24	88.64	90.53	92.05	93.31	94.36	95.25	96.02	96.70	97.29
	49.83	55.30	60.34	65.21	69.98	74.68	79.35	83.98	88.59	93.19	97.77	102.34	106.90	111.46
	14.06	16.07	17.12	17.79	18.25	18.59	18.85	19.05	19.22	19.35	19.46	19.55	19.63	19.70
c = .5	147.56	160.57	172.62	184.29	195.75	207.09	218.35	229.55	240.72	251.85	262.96	274.05	285.13	296.20
	70.29	80.34	85.60	88.94	91.26	92.96	94.25	95.27	96.09	96.75	97.31	97.77	98.17	98.52
	60.47	65.81	70.75	75.53	80.23	84.87	89.49	94.08	98.65	103.22	107.77	112.32	116.86	121.39
	15.61	17.26	18.08	18.57	18.90	19.13	19.31	19.44	19.54	19.63	19.70	19.75	19.80	19.84
c = .6	173.11	185.77	197.58	209.07	220.40	231.63	242.81	253.94	265.04	276.13	287.20	298.25	309.30	320.34
	78.06	86.32	90.38	92.83	94.49	95.65	96.53	97.19	97.72	98.14	98.48	98.76	98.99	99.19
	70.95	76.13	80.97	85.68	90.33	94.93	99.51	104.07	108.62	113.17	117.70	122.23	126.76	131.29
	16.81	18.12	18.73	19.08	19.31	19.47	19.58	19.67	19.74	19.79	19.83	19.86	19.89	19.91
c = .7	198.31	210.62	222.23	233.58	244.81	255.97	267.08	278.17	289.23	300.29	311.33	322.36	333.39	344.41
	84.05	90.60	93.64	95.40	96.55	97.33	97.91	98.35	98.67	98.93	99.14	99.30	99.44	99.55
	81.27	86.32	91.08	95.73	100.33	104.90	109.46	114.00	118.54	123.07	127.59	132.12	136.63	141.15
	17.71	18.72	19.16	19.41	19.57	19.68	19.75	19.80	19.85	19.88	19.90	19.92	19.94	19.95
c = .8	223.21	235.22	246.66	257.91	269.07	280.18	291.25	302.30	313.35	324.38	335.40	346.42	357.44	368.45
	88.55	93.61	95.82	97.06	97.84	98.38	98.75	99.02	99.23	99.39	99.51	99.61	99.69	99.76
	91.48	96.40	101.09	105.70	110.27	114.83	119.36	123.90	128.42	132.94	137.46	141.98	146.49	151.00
	18.38	19.14	19.46	19.63	19.73	19.80	19.85	19.88	19.91	19.93	19.95	19.96	19.97	19.97
c = .9	247.87	259.64	270.96	282.13	293.23	304.30	315.35	326.39	337.41	348.43	359.45	370.46	381.47	392.47
	91.88	95.70	97.28	98.14	98.66	99.01	99.25	99.42	99.55	99.65	99.73	99.78	99.83	99.87
	101.59	106.41	111.05	115.63	120.18	124.71	129.24	133.76	138.29	142.80	147.31	151.83	156.34	160.85

Table 11: Performance Measures for K = 240 and k = 20. (Entries corresponding to values of c greater than 3 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	4.12	6.93	9.15	11.16	12.98	14.59	16.13	17.65	18.93	20.34	21.58	23.04	24.04	25.28
	44.66	59.95	71.53	80.82	88.88	101.93	114.91	123.71	135.39	150.32	161.44	172.78	182.62	193.74
	12.87	21.66	28.60	34.87	40.55	45.59	50.41	55.15	59.16	63.56	67.44	72.02	75.12	78.99
	18.30	24.57	29.32	33.12	36.43	41.77	47.09	50.70	55.49	61.61	66.16	70.81	74.84	79.40
c = .2	7.21	10.60	13.07	15.21	17.10	18.56	19.97	21.38	22.59	23.67	24.49	25.45	26.32	27.25
	65.43	81.51	92.81	103.78	112.84	125.89	139.44	149.12	163.02	175.68	188.64	196.23	207.58	221.19
	22.53	33.12	40.83	47.54	53.45	58.01	62.42	66.82	70.58	73.96	76.52	79.53	82.24	85.15
	26.82	33.41	38.04	42.53	46.25	51.59	57.15	61.12	66.81	72.00	77.31	80.42	85.07	90.65
c = .3	10.10	14.09	16.82	18.76	20.41	21.80	23.10	24.12	25.13	26.01	26.84	27.52	28.13	28.74
	97.40	112.58	122.52	130.86	144.51	157.10	170.40	177.82	196.43	207.71	220.54	229.22	238.63	248.84
	31.57	44.02	52.57	58.63	63.79	68.14	72.20	75.38	78.53	81.29	83.87	86.00	87.92	89.81
	39.92	46.14	50.21	53.63	59.23	64.39	69.84	72.88	80.50	85.13	90.38	93.94	97.80	101.98
c = .4	13.24	17.31	19.92	21.83	23.32	24.52	25.53	26.39	27.13	27.78	28.36	28.88	29.36	29.80
	122.14	136.61	149.62	161.93	173.82	185.42	196.83	208.09	219.23	230.28	241.26	252.18	263.06	273.90
	41.38	54.11	62.26	68.22	72.87	76.63	79.78	82.46	84.78	86.81	88.63	90.27	91.75	93.13
	50.06	55.99	61.32	66.37	71.24	75.99	80.67	85.28	89.85	94.38	98.88	103.35	107.81	112.25
c = .5	15.89	19.92	22.35	24.07	25.36	26.38	27.21	27.90	28.48	28.99	29.43	29.82	30.16	30.48
	148.79	163.22	176.05	188.17	199.86	211.28	222.51	233.60	244.58	255.49	266.33	277.12	287.86	298.58
	49.67	62.24	69.86	75.21	79.24	82.43	85.02	87.18	89.01	90.58	91.96	93.17	94.26	95.25
	60.98	66.89	72.15	77.12	81.91	86.59	91.19	95.74	100.24	104.71	109.15	113.57	117.98	122.37
c = .6	18.30	22.13	24.34	25.84	26.94	27.78	28.46	29.01	29.46	29.85	30.18	30.47	30.72	30.95
	175.20	189.43	202.04	213.94	225.44	236.68	247.76	258.71	269.56	280.35	291.08	301.77	312.42	323.05
	57.18	69.16	76.07	80.75	84.18	86.82	88.93	90.64	92.07	93.28	94.31	95.21	96.01	96.71
	71.80	77.64	82.80	87.68	92.39	97.00	101.54	106.03	110.48	114.90	119.30	123.68	128.04	132.40
c = .7	20.43	23.99	25.95	27.24	28.16	28.85	29.39	29.82	30.17	30.47	30.71	30.93	31.11	31.27
	201.33	215.29	227.65	239.34	250.66	261.75	272.69	283.52	294.27	304.97	315.61	326.23	336.81	347.37
	63.85	74.98	81.10	85.12	87.99	90.15	91.83	93.18	94.29	95.21	95.98	96.64	97.22	97.73
	82.51	88.23	93.30	98.09	102.73	107.27	111.76	116.20	120.60	124.99	129.35	133.70	138.04	142.37
c = .8	22.31	25.54	27.25	28.33	29.09	29.65	30.08	30.41	30.68	30.91	31.09	31.25	31.38	31.50
	227.21	240.84	252.95	264.43	275.59	286.55	297.38	308.11	318.78	329.41	339.99	350.55	361.08	371.60
	69.72	79.82	85.14	88.54	90.91	92.65	93.99	95.04	95.89	96.58	97.16	97.65	98.07	98.43
	93.12	98.71	103.67	108.37	112.95	117.44	121.88	126.28	130.65	135.00	139.34	143.67	147.98	152.29
c = .9	23.93	26.82	28.28	29.18	29.80	30.25	30.59	30.85	31.05	31.22	31.36	31.47	31.57	31.65
	252.83	266.12	277.98	289.28	300.30	311.15	321.89	332.55	343.15	353.72	364.26	374.77	385.27	395.75
	74.79	83.81	88.37	91.20	93.13	94.53	95.58	96.40	97.05	97.57	98.00	98.35	98.66	98.92
	103.62	109.06	113.93	118.56	123.07	127.52	131.92	136.29	140.64	144.97	149.29	153.60	157.90	162.19

Table 12: Performance Measures for K = 240 and k = 32. (Entries corresponding to values of c greater than 3 are asymptotic values.)

Significance Level (α)

	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70
c = .1	1.82	2.88	3.71	4.39	4.98	5.50	5.97	6.39	6.78	7.14	7.48	7.80	8.11	8.40
	21.82	28.38	34.71	40.89	46.98	53.00	58.97	64.89	70.78	76.64	82.48	88.30	94.11	99.90
	18.18	28.83	37.06	43.90	49.79	54.99	59.65	63.90	67.80	71.43	74.83	78.04	81.09	84.02
	17.60	22.89	27.99	32.98	37.89	42.74	47.55	52.33	57.08	61.81	66.52	71.21	75.89	80.57
c = .2	3.45	4.76	5.63	6.28	6.79	7.22	7.58	7.89	8.16	8.40	8.61	8.81	8.99	9.15
	35.46	42.26	48.63	54.78	60.79	66.72	72.58	78.39	84.16	89.90	95.61	101.31	106.99	112.65
	34.55	47.58	56.27	62.77	67.93	72.18	75.77	78.86	81.57	83.97	86.12	88.08	89.87	91.53
	28.59	34.08	39.22	44.18	49.03	53.81	58.53	63.21	67.87	72.50	77.11	81.70	86.28	90.85
c = .3	4.93	6.24	7.02	7.57	7.98	8.30	8.55	8.77	8.95	9.10	9.23	9.35	9.46	9.55
	48.93	55.74	62.02	68.07	73.98	79.80	85.55	91.27	96.95	102.60	108.24	113.85	119.46	125.05
	49.34	62.38	70.22	75.67	79.76	82.96	85.54	87.68	89.48	91.02	92.35	93.53	94.57	95.52
	39.46	44.95	50.02	54.89	59.66	64.35	69.00	73.60	78.18	82.74	87.29	91.82	96.34	100.85
c = .4	6.18	7.36	8.01	8.43	8.74	8.97	9.14	9.28	9.40	9.50	9.58	9.65	9.71	9.76
	62.18	68.86	75.01	80.93	86.74	92.46	98.14	103.79	109.40	115.00	120.58	126.15	131.71	137.26
	61.83	73.58	80.07	84.32	87.36	89.65	91.43	92.85	94.02	94.98	95.80	96.49	97.09	97.63
	50.15	55.53	60.49	65.27	69.95	74.57	79.15	83.70	88.23	92.74	97.24	101.73	106.22	110.70
c = .5	7.19	9.18	8.69	9.00	9.22	9.38	9.49	9.59	9.66	9.72	9.77	9.81	9.85	9.87
	75.19	81.68	87.69	93.50	99.22	104.88	110.49	116.09	121.66	127.22	132.77	138.31	143.84	149.37
	71.87	81.78	86.85	90.01	92.18	93.76	94.95	95.87	96.61	97.20	97.69	98.10	98.45	98.74
	60.63	65.87	70.71	75.40	80.01	84.58	89.11	93.62	98.11	102.60	107.07	111.54	116.00	120.46
c = .6	7.97	8.76	9.14	9.37	9.52	9.63	9.70	9.76	9.81	9.84	9.87	9.90	9.92	9.93
	87.97	94.26	100.14	105.87	111.52	117.13	122.70	128.26	133.81	139.34	144.87	150.40	155.92	161.43
	79.66	87.62	91.43	93.70	95.20	96.26	97.04	97.63	98.08	98.44	98.73	98.97	99.17	99.33
	70.94	75.02	80.76	85.38	89.94	94.46	98.95	103.44	107.91	112.37	116.83	121.29	125.74	130.19
c = .7	8.55	9.17	9.45	9.61	9.71	9.78	9.83	9.86	9.89	9.91	9.93	9.94	9.96	9.97
	100.55	106.67	112.45	118.11	123.71	129.28	134.83	140.36	145.89	151.41	156.93	162.44	167.96	173.46
	85.52	91.70	94.48	96.06	97.08	97.77	98.27	98.64	98.92	99.14	99.31	99.44	99.56	99.65
	81.09	86.02	90.68	95.25	99.76	104.26	108.73	113.20	117.65	122.11	126.56	131.00	135.45	139.89
c = .8	8.98	9.45	9.65	9.76	9.82	9.87	9.90	9.92	9.94	9.95	9.96	9.97	9.98	9.98
	112.98	118.95	124.65	130.26	135.82	141.37	146.90	152.42	157.94	163.45	168.96	174.47	179.98	185.48
	89.84	94.49	96.47	97.56	98.23	98.68	98.99	99.22	99.39	99.52	99.62	99.70	99.76	99.81
	91.12	95.93	100.52	105.05	109.53	114.01	118.47	122.92	127.37	131.82	136.26	140.70	145.14	149.58
c = .9	9.30	9.64	9.78	9.85	9.89	9.92	9.94	9.96	9.97	9.97	9.98	9.98	9.99	9.99
	125.30	131.14	136.78	142.35	147.89	153.42	158.94	164.46	169.97	175.47	180.98	186.48	191.99	197.49
	92.96	96.38	97.77	98.50	98.94	99.22	99.42	99.56	99.66	99.73	99.79	99.84	99.87	99.90
	101.04	105.76	110.30	114.80	119.27	123.73	128.18	132.63	137.07	141.51	145.95	150.39	154.83	159.27

Table 13: Performance Measures for K = 120 and k = 10. (All entries are asymptotic values.)

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In simulation models involving many factors (input variables), usually relatively few are important. Often it is desired to "screen" out these important factors using a limited number of computer runs. This paper examines the performance of a two-stage screening procedure based on random balance and Plackett-Burman designs.

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